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Multi-storey, multi-bay buildings with composite steel-deck floors under human-induced loads: The human comfort issue

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ABSTRACT

Inadequate dynamic response of steel and concrete composite floors lead to comfort problems when human rhythmic activities are carried out. The major aspects governing this problem are discussed in this paper. Structural models representative of common buildings were loaded with two dynamic load models, and an evaluation of their behaviour focusing on the numerically predicted peak accelerations carried out. Their critical analysis and comparison to limiting values proposed in the literature allowed to establish conclusions concerning the suitability of this structural solution, and the influence of the span and load pattern when the issue of annoying vibrations is dealt with.

1. Introduction

Human loads constitute a large portion of the live loads acting on the floors of offices and commercial and residential building structures, and they act frequently as dynamic loads, which may lead to a degree of discomfort from excessive vibrations. The issue of the negative effects of excessive vibrations on human comfort is a long-standing concern in structural design; Treggold [1] addressed this issue in 1828 by stating that large span beams should have a minimum height to prevent unacceptable vibrations when dynamically loaded by users walking on the pavement.

Today's trends in architectural and structural engineering design favour slender structures; larger spans, creating large open spaces; non-conventional shapes and structural solutions; more resistant materials, such as high strength concrete or larger steel grades; and steel and concrete composite structures. Additionally, more sophisticated analytical models contribute to enabling the design of irregular and economically optimised structural shapes.

As a result of these trends, smaller sections are used in structural elements, such as lighter and more flexible beams and slabs, with implications on the natural frequencies and dynamical response. This is the case for the composite steel-deck floors associated with steel beams, a very common solution for buildings and are addressed in the current paper. This structural system often presents low natural frequencies, not far from those induced by human rhythmic actions, such as walking, dancing, or aerobics [2], and therefore may suffer from vibration problems affecting the comfort of users, justifying the verification of the excessive vibration limit state. In fact, this limit state may become a key issue for the serviceability of these structures, together with the more common deformation limit state and the ultimate limit state verifications.

The analysis of the structural vibrations should include a dynamic analysis and a comparison of the predicted accelerations to the human allowances related to comfort, although simplified criteria may often be used based on the floor flexibility or the natural frequency.

The issue of the economical design of service buildings with steel columns and steel and concrete composite floors was addressed by Costa [3], who performed a comparative study for varying steel grades and the spans, where each structure was designed according to Eurocode 3 [4] and Eurocode 4 [5]. The design coped with the verifications of the ultimate limit states and the deformation limit state under service loads, and the steel weights for each type of element and for the whole structure were determined. In addition, Costa presented curves relating the various parameters and obtained trends for determining the optimum (economical) span [3].

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The current paper reports the results of a study conducted at the University of Coimbra [6], which focused on the excessive vibration limit state in the design of service buildings with composite floors. The paper is based on similar structural layouts and designs used in [3] and associated with a real structure, in the scope of a project involving the construction of 50,000 modular buildings, including residential and general use buildings. In that project, due to the large number of buildings, possible geometries and structural layouts, a parametric optimization of the structure in the predesign phase was carried out by varying the materials, the structural layout and the spans [3]. A comparative assessment of these structures dynamic response was carried out in a further step [6].

Dynamic loadings resulting from human-induced rhythmic activities related to aerobics is applied to each structural model and the natural frequencies, the corresponding vibration modes, and the expected peak accelerations are evaluated and compared to design recommendations [7,8]. Two simultaneous types of occupancies are considered in the same building: aerobics and general services use. The verification of the excessive vibration limit state when aerobics is being performed in parts of the structures is presented and discussed, for the parts of the building where this activity is being performed and for the neighbour parts occupied by general services activities.

2. Human-induced dynamic loads

The issue of floor vibrations induced by human rhythmic activities, such as walking, running, jumping or aerobics, is quite complex because the dynamical excitation characteristics generated during these activities are directly related to the individual body adversities and the specific way in which each human being executes a certain rhythmic task. These aspects prevent an easy mathematical or physical characterisation of this phenomenon [9].

Humans have always analysed the most apparent distinctions of the various activities they perform. However, the fundamental mechanical analysis of these tasks was not possible before significant developments in mechanical science. Initially, human motion received an incipient attention from researchers, such as Borelli in 1679 [10] and the Weber brothers in 1836 [11]. The first pioneer in this field was Otto Fischer, a German mathematician who conducted the first study comprising a comprehensive evaluation of the forces involved in human motion in 1895.

To determine the dynamical behaviour of floor structural systems subjected to excitations from human activities, various studies have attempted to evaluate the magnitude of these rhythmic loads. The next stage of this research line was the development of a loading platform by Elftman [10] that enabled the determination of the ground reactions to the foot forces associated with walking. A typical force platform is comprised of an approximately 1-m^2 steel plate supported by four small columns at the plate midsides. Load cells were installed at each of the columns to detect the magnitude of the load variation at these points. These results allowed the magnitude and direction of the forces transmitted to the supporting surface, denominated ground reaction forces, to be determined.

Rainer et al. [11] also contributed to this investigation, developing more sophisticated load platforms that recorded the ground reaction forces coming from the foot forces associated with human motion. Ebrahimpur et al. [12] developed a 14.2-m-long, 2-m-wide platform designed to record the actions from the walking motions of one, two, or four individuals.

Another load model used to represent the walking motion forces is expressed as a function of tests recording the heel impact on the floor. This load type, considered to be the main excitation source during the human walking motion, produces a transient response, i.e., the system is excited by an instantaneous application of force. Its graphical representation was provided by Ohmart [13] in denominated heel-drop experiments, in which the individual drops his or her heel onto the floor after elevating it to a height corresponding to the individual's weight.

The heel-drop test was also conducted by Murray and Hendrick [14] in different building types. A 0.84 kN impact force was measured by a seismograph in nine church ceremonial rooms, three slabs located on the top storey of a shopping mall, two balcony slabs at a hotel, and one slab located on the second storey of a commercial building. The structural dynamic responses of the investigated structural systems in terms of force amplitudes, frequencies, and damping could then be determined.

A significant contribution to this field was made in Brazil by Alves [15] and Faísca [9] based on experiments made with a group of volunteers acting on a concrete platform. These tests enabled the development of approximated descriptions of the loads induced by human activities, such as jumping, aerobics, football (soccer), and audience responses to rock concerts. These tests were executed over two concrete platforms, one rigid and the other flexible, both of which were placed on movable supports, which allowed the structure stiffness to vary, enabling the investigation of the human rhythmic load over rigid or flexible structures. The experimental results and the obtained analytical model led to the development of load functions for synchronous and asynchronous activities that could be used in structural designs intended for stadiums and other similar structures.

Although the floor vibration problems induced by human activities have grown significantly over the last few decades, it should be stressed that this research field is not new. In 1828, Tredgold [1] proposed design criteria to avoid, or minimise, undesirable effects related to floor vibrations by increasing the beam heights used in large span structures. Since then, numerous design criteria have been proposed all over the world in an attempt to establish vibration limits that do not compromise human comfort.

Reiher and Meister [16] proposed a scale enabling the description of human perception and acceptable levels for continuous vibrations. The scale was calibrated in terms of the frequencies and amplitudes of the displacements based on the results of tests in which a group of standing individuals was subjected to continuous vibrations within a frequency range of 1–100 Hz and amplitudes ranging from 0.01 to 10 mm. Lezen [17] evaluated the dynamical response of two floors in the laboratory and 46 different building floors designed for offices, churches, classrooms, *etc.*, concluding that the original Reiher and Meister [16] scale could be modified for use in floors with damping ratios less than 5%. Wiss and Parmalee [18] presented a study in which a group of 40 individuals was subjected to a specific load function designed to simulate the vibrations usually present in building structures. The aim of this study was to experimentally investigate the human reaction to transient vertical vibrations in terms of frequency, displacement, and damping.

Murray [19] classified the human vibration perception into four categories: unnoticeable by occupants; noticeable but does not disturb the occupants; noticeable and disturbs the occupants; compromises the occupants' security. These categories were established based on 100 heel-drop tests performed on steel and concrete composite floors.

Allen et al. [20] proposed minimum values (greater than 6 Hz) for the natural frequencies of structures, evaluated according to the type of occupancy and their main characteristics. These values were based on the dynamical load values produced by human rhythmic activities, such as dancing or aerobics, and the limit acceleration values associated with these activities.

The determination of a minimum thickness for rectangular slabs subjected to harmonic loads induced by human dynamic actions was also presented by Pasquetti et al. [21]. These authors developed charts to aid structural designers in evaluating the dynamical responses of residential building slabs subjected to rapid walking or other rhythmic activities.

Batista and Varela [22] experimentally determined that the problems related to dynamical excitations produced by human rhythmic actions are more pronounced and frequent in continuous slab panels that present coupled vibration modes, such as composite slabs, waffled and grillage slab systems, or precast concrete slabs. Batista and Varela [22] also verified that a 60% increase in the original slab thickness or the use of light partition panels was not an efficient solution for excessive vibrations. A good solution was the use of synchronised dynamical attenuators capable of reducing the maximum amplitude of the dynamical response for a specific natural vibration frequency.

Paula and Queiroz [23] presented a study of a composite structure (steel beams and concrete slab) designed for static loads that was subsequently subjected to human rhythmic activities. The dynamical load representative of the rhythmic activity was simulated in a finite element model by harmonic loads where the main excitation frequency and some of its higher harmonics were considered. The structure natural frequency results obtained from this model were then compared to experiments on a similar structure. A proposal for strengthening the structure for its new use was presented based on an evaluation of the new levels of accelerations and stresses present in the structure.

Numerical studies on the dynamic behaviour of composite steel-deck floors subjected to human-induced vibrations were published by Williams and Waldron [24], Allen [25], Santos da Silva et al. [26], and El-Dardiry and Ji [27].

Design recommendations based on some of these research findings are provided by the International Standard Organisation (ISO 2631-2) [7], the American Institute of Steel Construction (AISC) [8], and the Steel Construction Institute (SCI) [28], which provide acceleration limits for the human perceptibility for various occupancies under different human-induced dynamic loads, and a comprehensive review of vibration serviceability criteria for floor structures was presented by Ebrahimpour and Sack [29].

El-Dardiry and Ji [27] observed that the mode shapes of the panels in a multi-panel floor are different and complicated, but the mode shape of each panel is either concave or convex. They developed two equivalent flat plate models (for isotropic and orthotropic flat plates) for predicting the dynamic behaviour of composite floors, using the equivalence of the maximum displacement of a sophisticated three-dimensional composite panel model. The authors also parametrically studied the effects of boundary conditions, loading conditions, shear modulus, and steel sheet characteristics on the equivalent floor models [27].

The dynamic characteristics of a multi-panel floor system under human-induced loads using finite element techniques were investigated by de Silva and Thambiratnam [30]. Loading models with variable parameters of intensity, foot contact ratio, frequency, and damping were developed and applied as pattern loads. The floor panel response in terms of deflection and acceleration was evaluated and used to assess the panel suitability for different occupancies. The authors have highlighted the occurrence of multimodal vibration and the relevance of applying pattern loads to capture these modes, as they also can cause discomfort and excessive deflections in floor panels [30].

Other studies focusing on the floor vibration induced by dance-type loads were presented by Ellis and Ji [31,32] that developed an analytical solution for this problem and made some experimental and numerical studies. The same authors focused on the problem of the response of structures with loads generated by jumping crowds, including the effects aroused by the fact that in a crowd not everyone is jumping in perfect synchronization, but includes people dancing and clapping, and some people stationary [33,34].

The potential benefits of incorporating active vibration control in the design of building structures to satisfy vibration serviceability limits for human-induced vibrations in floor structures was studied by Hudson and Reynolds [35,36].

3. Characterisation of dynamic loading induced by human rhythmic activities

The characterisation of the dynamic loads generated by human rhythmic activities must include the specific characteristics of each individual performing the same activity and the existence of external excitation. In general, human live loads are classified into two broad categories: in situ and moving. Periodic jumping due to music, the sudden standing of a crowd, and random in-place movements are examples of in situ activities. Walking, marching, and running are examples of moving activities.

Numerous investigations aiming at establishing parameters to describe such dynamic actions were made in the past [8,9,37–40], contributing to the design of safe and comfortable steel and concrete composite floors that support motion.

Two different load models were proposed by the authors of some of these previous studies to incorporate the dynamic effects induced by human rhythmic activities on steel–concrete composite floors, as presented in the following sections.

3.1. Loading model I (LM-I)

The loading generated by human activities may be described as a Fourier series, which incorporates a static part due to the individual's weight and a dynamic part due to the dynamic load [8,9,37–40]. The dynamic analysis is performed by equating one of the activity harmonic frequencies to the floor fundamental frequency, leading to a resonance condition.

It is worth mentioning that one of the main contributions of this paper is related to the modelling of the human rhythmic dynamic actions. The dynamic actions generated by human rhythmic activities such as stimulated and non-stimulated jumping, aerobics, football crowds, spectators in concerts and dancing are included in the loading model I (LM-I), see Eq. (1). It is important to emphasise that the impact coefficient, K_p, and the phase coefficient variation, CD, for human activities used in this investigation were obtained based on a long series of experimental tests and probabilistic analyses. Relevant variations which lead to the reduction of the dynamic loading on the floor, such as phase lags between the individuals and change of rhythm during the activity are already embedded in these coefficients, see Eq. (1). In this dynamic loading model, three harmonics were considered to represent the load associated with human rhythmic activities, see Figs. 1–3, Tables 1 and 2.

This way, Fig. 2 and Table 1 illustrate the phase coefficient variation, CD, for human activities studied by Faisca [9], considering a certain number of individuals and later extrapolated for a large number of people, based on probabilistic analyses. Table 2 presents the experimental parameters used for representing human rhythmic activities, and Fig. 3 presents some examples of the dynamic actions related to human rhythmic activities investigated in this work, illustrating the three harmonics considered in this loading model, when a frequency domain analysis was performed.

$$F(t) = \mathrm{CD}\left\{K_p P \sum \left(0.5 - 0.5\cos\left(\frac{2\pi i}{T_c}t\right)\right)\right\} \quad \text{when } t \le T_c$$
$$F(t) = 0 \quad \text{when } T_c \le t \le T \tag{1}$$

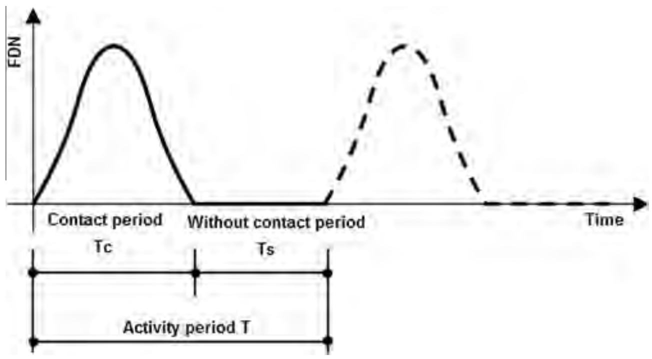


Fig. 1. Representation of the dynamic loading induced by human rhythmic activities.

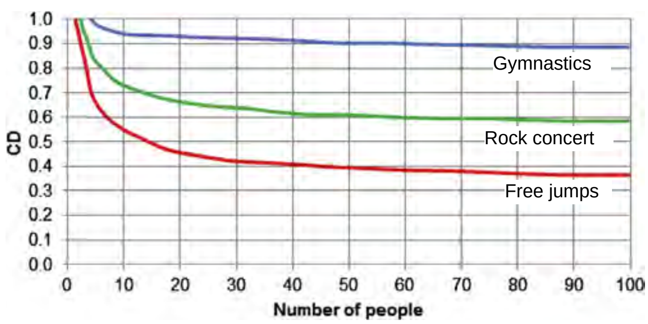


Fig. 2. Variation of the phase coefficient CD for human rhythmic activities [9].

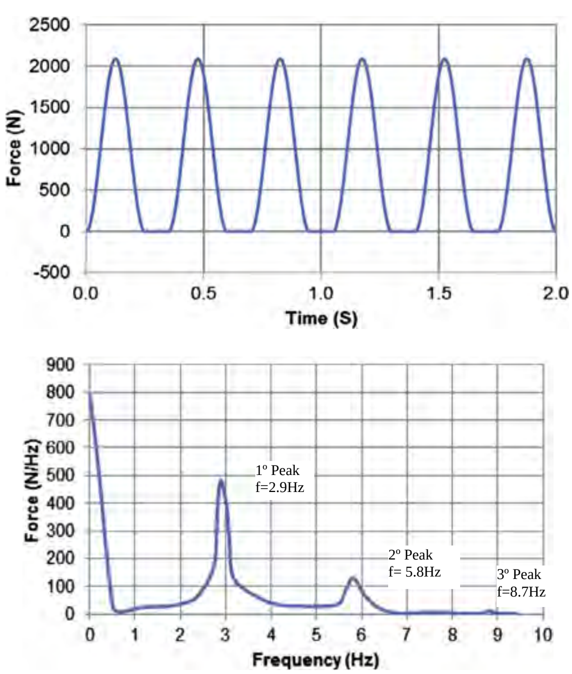


Fig. 3. Dynamic loading induced by human rhythmic activities (LM-I: aerobics). $T = 0.35$ s, $T_c = 0.25$ s, $K_p = 2.78$, CD = 1.0.

Here,
$F(t)$: dynamic load (N);
CD: phase coefficient;
K_p: impact coefficient;
P: person's weight (800 N [37,38,40]);
T_c: activity contact period (s);
T: activity period (s);
t: time (s).

Table 1
Numeric values adopted for the phase coefficient CD [9].

Number of people	Aerobics class	Free jumps
1	1	1
3	1	0.88
6	0.97	0.74
9	0.96	0.70
12	0.95	0.67
16	0.94	0.64
24	0.93	0.62
32	0.92	0.60

Table 2
Experimental parameters used for human rhythmic activities [9].

Activity	T (s)	T_c (s)	K_p
Free jumps	0.44 ± 0.15	0.32 ± 0.09	3.17 ± 0.58
Aerobics class	0.44 ± 0.09	0.34 ± 0.09	2.78 ± 0.60
Shows	0.37 ± 0.03	0.37 ± 0.03	2.41 ± 0.51

3.2. Loading model II (LM-II)

This dynamic loading model [8] can be represented by the load static fraction related to the individual's weight and a combination of harmonic forces with frequencies that are multiples or harmonics of the basic frequency of the force repetition, e.g., the step frequency, f_s, for human rhythmic activities. The model considers a spatial and temporal variation of the dynamic action over the structure, and the time-dependent repeated force can be represented by the Fourier series in Eq. (2).

$$F(t) = P[1 + \sum \alpha_i \cos(2\pi i f_s t + \phi_i)] \tag{2}$$

Here,
$F(t)$: dynamic load (N);
P: person's weight (800 N [37,38,40]);
α_i: dynamic coefficient for the harmonic force;
i: harmonic multiple ($i = 1,2,3,\ldots,n$);
f_s: walking step frequency (Hz);
ϕ: harmonic phase angle;
t: time (s).

Table 3
Step frequency and dynamic coefficients [8].

Harmonic i	Aerobics class	
	Step frequency f_p (Hz)	Dynamic coefficients α_i
1	2.0–2.75	1.5
2	4.0–5.5	0.6
3	6.0–8.25	0.1

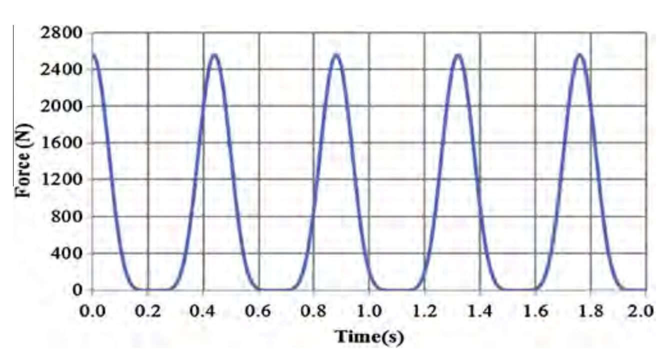


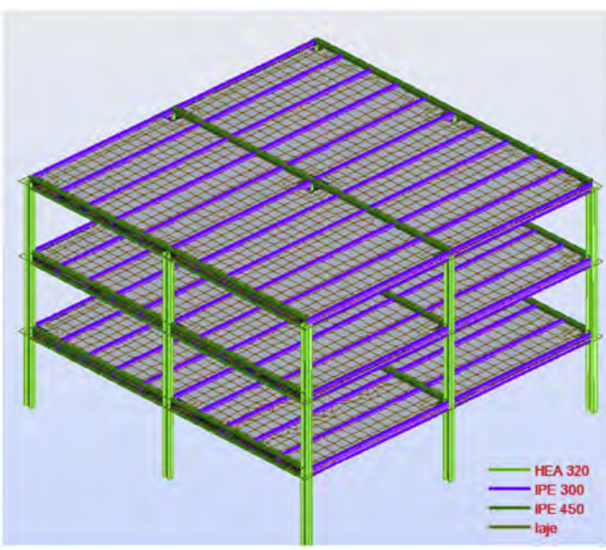
Fig. 4. Dynamic loading induced by human rhythmic activities (LM-II: aerobics). $f_p = 2.27$ Hz; $\phi = 0$; $\alpha_1 = 1.5$; $\alpha_2 = 0.6$; $\alpha_3 = 0.1$.

Three harmonics are considered to represent the dynamic load associated with human rhythmic activities [8]. Table 3 shows the step frequency and dynamic coefficients used in this model. Additionally, the phase angles were assumed to be zero. Fig. 4 illustrates a dynamic loading induced by human rhythmic activities.

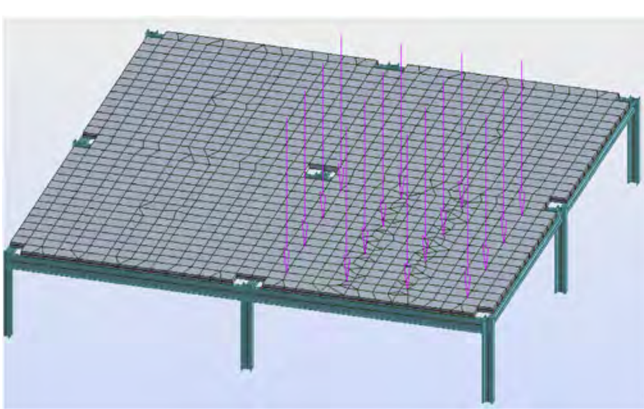
4. Structural features

The investigated structural models correspond to the buildings with steel–concrete composite floors, based on real structures, described in [3], with two types of occupancies: aerobics and service use. These buildings may experience these two different occupancies on different floor panels simultaneously. The models are composed of composite girders and a composite steel-deck floor, with the structural layout shown in Figs. 5 and 6, supported by steel columns and subjected to human rhythmic loads in possibly alternate panels.

The spans take values of 4 × 4 m, 5 × 5 m, 5.7 × 5.7 m, 6.7 × 6.7 m, 8 × 8 m, and 10 × 10 m, with the total area varying from 16 m^2 to 100 m^2 per panel, as illustrated in Table 4, where steel sections for the different structures are shown as well. A S355 steel grade (yield stress f_y = 355 MPa and Young modulus E = 200 GPa) was adopted in the present study for the steel elements, selected from the different steel grades studied in [3] as the grade leading to the lightest structures. An orthotropic 100-mm-thick composite steel-deck floor [3] was adopted in the design, with C30/37 concrete class according to Eurocode 2 [41] (characteristic compressive strength f_{ck} = 30 MPa and Young modulus E_{cm} = 33 GPa) and a total mass of 346 kg/m^2.

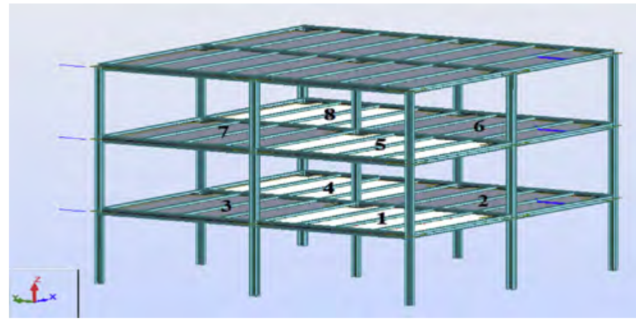


(a) Finite element mesh (illustration for a 8 m x 8 m structure).

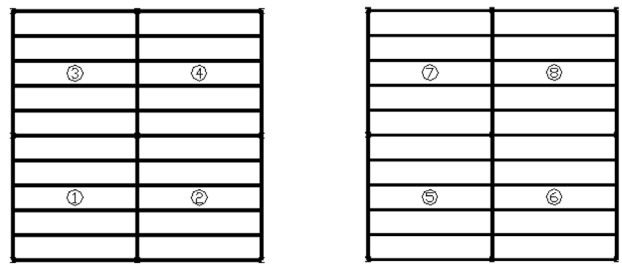


(b) Load distribution at the 8 m x 8 m panel structure: first storey.

Fig. 5. Finite element mesh and load distribution at the 8 × 8 m panel structure.



(a) Building structure: general view and panel numbers.



Storey 1: z = 3 m. Storey 2: z = 6 m.

(b) Observation points.

Fig. 6. Structure geometry and observation points.

Table 4
Element sections for the different structural models.

Span (m × m)	Main girders	Secondary girders	Columns
4 × 4	IPE 160	IPE 160	HEA 160
5 × 5	IPE 220	IPE 200	HEA 220
5.7 × 5.7	IPE 270	IPE 220	HEA 220
6.7 × 6.7	IPE 330	IPE 240	HEA 260
8 × 8	IPE 450	IPE 300	HEA 320
10 × 10	IPE 600	IPE 360	HEA 450

The structure was loaded by its self-weight and other dead loads, such as floors, walls, insulations and roofs (3.5 kN/m^2), and considered to have a quasi-permanent part (30%) of the characteristic live load of 3 kN/m^2 on the panels that are not loaded dynamically.

The dynamic live load is based on the occupancy of one person for each 4.0 m^2 (0.25 person/m^2), according to [40], and therefore corresponding to the effect of 4–25 individuals practising aerobics, as indicated in Table 6. The load distribution was considered to be symmetrically centred on the slab panels, as depicted in Fig. 5b. It is also assumed that one person's weight is equal to 800 N (0.8 kN) [37,38,40].

Ten different dynamic load cases, corresponding to the dynamical loading of an aerobics class, were applied, as shown in Fig. 5b, at the panels indicated in Table 5 and illustrated in Fig. 6. In this way, different combinations representing the effect of loading a single panel, all panels, or alternate panels are considered. In addition because this procedure is repeated for each storey, the effect of the rhythmic activity on the other storey may be assessed as well.

Table 5
Definition of the dynamic loading in the structure (**X**: loaded; **0**: not loaded).

Load case	Floor panels (see Figs. 5 and 6)							
	1	2	3	4	5	6	7	8
1	**X**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
2	**X**	**X**	**0**	**0**	**0**	**0**	**0**	**0**
3	**X**	**0**	**X**	**0**	**0**	**0**	**0**	**0**
4	**X**	**0**	**0**	**X**	**0**	**0**	**0**	**0**
5	**X**	**X**	**X**	**X**	**0**	**0**	**0**	**0**
6	**0**	**0**	**0**	**0**	**X**	**0**	**0**	**0**
7	**0**	**0**	**0**	**0**	**X**	**X**	**0**	**0**
8	**0**	**0**	**0**	**0**	**X**	**0**	**X**	**0**
9	**0**	**0**	**0**	**0**	**X**	**0**	**0**	**X**
10	**0**	**0**	**0**	**0**	**X**	**X**	**X**	**X**

Table 6
Number of people at each panel for the different structural systems.

Composite floors	Investigated spans (see Figs. 5 and 6)					
	10×10	8×8	6.7×6.7	5.7×5.7	5×5	4×4
Area (m^2)	100	64	44.89	32.49	25	16
Number of people	25	16	12	9	7	4

Modeling the whole structure including the columns and the non-loaded floor makes the finite element analysis more realistic, since simply or fully supporting the beam nodes where the columns intersect would not lead exactly to the same results. In addition, this procedure allows the assessment of the human comfort in the slab panels of the non-loaded floor.

The investigated structural models were evaluated and the ultimate and serviceability limit states were properly considered, based on design codes provisions [4,5,41]. After that, the steel–concrete composite floors dynamic response, in terms of peak accelerations values, was obtained for panels 1–8 (Fig. 6) to verify its behaviour and the influence of the dynamic loading on the adjacent slab floors (see Figs. 5 and 6).

5. Finite element modelling and structural damping

The structures were analysed using the Robot Structural Analysis Professional [42] program with a computational model developed using the typical mesh refinement techniques present in finite element simulations and implemented in the Robot program [42].

Both materials (steel and concrete) were considered as having an elastic behaviour, and complete interaction between the concrete slab and steel beams was assumed in the analysis. In this way, the numerical model coupled all the nodes between the beams and the composite slab to prevent the occurrence of any slip. An illustrative finite element model for a representative span is illustrated in Figs. 5 and 6.

In these computational models, all "I" or "H" steel sections, related to beams and columns, respectively, were represented by three-dimensional beam elements with tension, compression, torsion, and bending capabilities. These elements have six degrees of freedom at each node: translations in the nodal *x*, *y*, and *z* directions and rotations about the *x*, *y*, and *z* axes.

The orthotropic reinforced concrete slab was represented by four-node square and three-node triangular shell finite elements with bending, shear, and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal *x*, *y*, and *z* directions and rotations about the nodal *x*, *y*, and *z* axes.

In this investigation, structural damping was considered according to the Rayleigh proportional damping formulation [43]. The composite floors structure damping matrix is defined by the parameters α and β, determined as a function of the damping modal coefficient. According to this formulation [43], the structural system damping matrix is proportional to the mass and stiffness matrixes, as shown in Eq. (3):

$$[\mathrm{C}] = \alpha[\mathrm{M}] + \beta[\mathrm{K}] \tag{3}$$

This expression may be rewritten in terms of the modal damping coefficient and the natural frequency, leading to Eq. (4):

$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \tag{4}$$

Here, ξ_i is the modal damping coefficient and ω_i is the natural frequency associated with mode shape "*i*". Isolating the Eq. (4) parameters α and β for two natural frequencies ω_{01} and ω_{02}, adopted according to the relevance of the corresponding vibration mode for the structural system dynamic response, generates

$$\beta = \frac{2(\xi_2\omega_{02} - \xi_1\omega_{01})}{\omega_{02}\omega_{02} - \omega_{01}\omega_{01}} \tag{5}$$

$$\alpha = 2\xi_1\omega_{01} - \beta\omega_{01}\omega_{01} \tag{6}$$

With two natural frequency values, it is possible to evaluate the parameters α and β described earlier using Eqs. (5) and (6). The reference frequencies ω_{01} and ω_{02} are generally taken as the extreme frequencies of the structure spectrum. In this paper, the adopted frequency ω_{01} is the structure's fundamental frequency, and the considered frequency ω_{02} is the system's 2nd natural frequency. The modal damping coefficient adopted in this investigation is equal to 0.01 (ξ = 1%) [8,37,40].

Based on the Rayleigh proportional damping formulation [43], the parameters α and β used in this analysis for the investigated structural models were α = 0.49153 and β = 0.00020 (4×4 m); α = 0.50329 and β = 0.00019 (5×5 m); α = 0.46936 and β = 0.00021 (5.7×5.7 m); α = 0.30220 and β = 0.00033 (6.7×6.7 m); α = 0.34521 and β = 0.00028 (8×8 m) and α = 0.27989 and β = 0.00035 (10×10 m).

6. Dynamic analysis

For practical purposes, a linear time-domain analysis was performed throughout this investigation. This section presents the evaluation of the steel–concrete composite floors vibration levels when subjected to human rhythmic activities.

The steel–concrete composite floor dynamic response was determined through the analysis of its natural frequencies and peak accelerations. The results of the dynamic analysis were obtained from an extensive analysis [6] based on the finite element method using the Robot Structural Analysis Professional [42] program.

To quantitatively and qualitatively evaluate the obtained results according to the proposed methodology, the steel–concrete composite floor peak accelerations were calculated and compared to the limiting values of design recommendations [7,8]. This comparison was made to access a possible occurrence of unwanted excessive vibration levels and human discomfort.

6.1. Natural frequencies and mode vibrations

Based on the previously referred analysis, the composite floor's natural frequencies were determined, as presented in Fig. 7 and Table 7. The floor vibration modes with beam spans of 8 m are illustrated in Fig. 8.

It must be emphasised that there was a very good agreement between the finite element natural frequencies values, as presented in Table 7, and the frequencies calculated using the

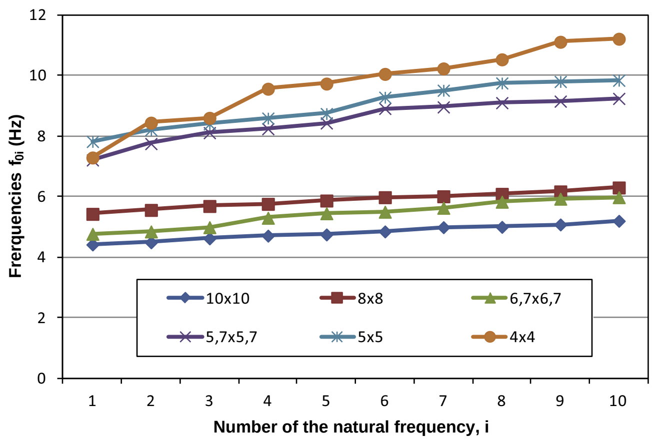


Fig. 7. Variation of the fundamental frequency for the studied structural models.

procedures proposed by Murray et al. [8]. This fact validates the numeric model presented herein as well as the results and conclusions obtained throughout this work.

The numerical results presented in Fig. 7 and Table 7 clearly indicate that the structural system's stiffness decreases with increasing span, reducing, as expected, the composite floor's natural frequencies. These results also indicate that when the floor span increases, some of these structures may become vulnerable to low forcing frequencies and undesirable vibrations.

6.2. Human comfort assessment – accelerations

The present investigation proceeded with the evaluation of the composite floor performance in terms of vibration serviceability due to human rhythmic activities in the form of aerobics. The first step of this procedure concerned the determination of the peak accelerations of the steel–concrete composite floors. The peak accelerations (a_p in mm/s^2) were determined based on the developed finite element models. These accelerations were then compared to results supplied by design criteria [7,8].

The two dynamic loading models previously described were applied to the composite floors to determine the peak acceleration considering the variation of the beam spans from 4 m to 10 m (see Figs. 5 and 6).

Based on the experimental results obtained by Faísca [9] for aerobics, the composite floors dynamic behaviour was evaluated using the LM-I [see Eq. (1)], keeping the human rhythmic activity period, T, equal to 0.44 s ($T = 0.44$ s); the contact period with the composite floor, T_c, equal to 0.34 s ($T_c = 0.34$ s); the period without contact with the floor, T_s, equal to 0.10 s ($T_s = 0.10$ s); and the impact coefficient value, K_p, equal to 2.78 ($K_p = 2.78$). When the LM-II [see Eq. (2)] was considered in the analysis, three harmonics were used to represent the dynamic excitation according to Table 3 [8], and the phase angles were assumed to be equal to zero.

Figs. 9 and 10 illustrate the dynamic response (accelerations: time domain) related to the investigated panels (see Fig. 6) when 16 individuals are performing aerobics on the composite floor (see Fig. 5), considering the complete interaction between the concrete slab and steel beams.

In Fig. 9, the composite floor dynamic response is presented based on the two investigated loading models, LM-I and LM-II (see Eqs. (1) and (2) and Figs. 3 and 4, respectively). In this figure, the dynamic response was evaluated on the excited panel of the 8 × 8 m composite floor, as presented in Fig. 5.

It can be concluded that the LM-II generates higher peak accelerations when compared with those obtained with the LM-I, as illustrated in Fig. 9. In fact, in the LM-II mathematical representation, the phase coefficient CD is not considered in its formulation, and this model overestimates the composite floor maximum accelerations. The LM-II does not incorporate any variations which lead to the reduction of the dynamic loading on the floor (phase lags between the individuals and change of rhythm during the activity). In this situation, the dynamic loads are in phase on the floor and the peak acceleration values are overestimated and not realistic. This fact was confirmed in several other loading cases in this study.

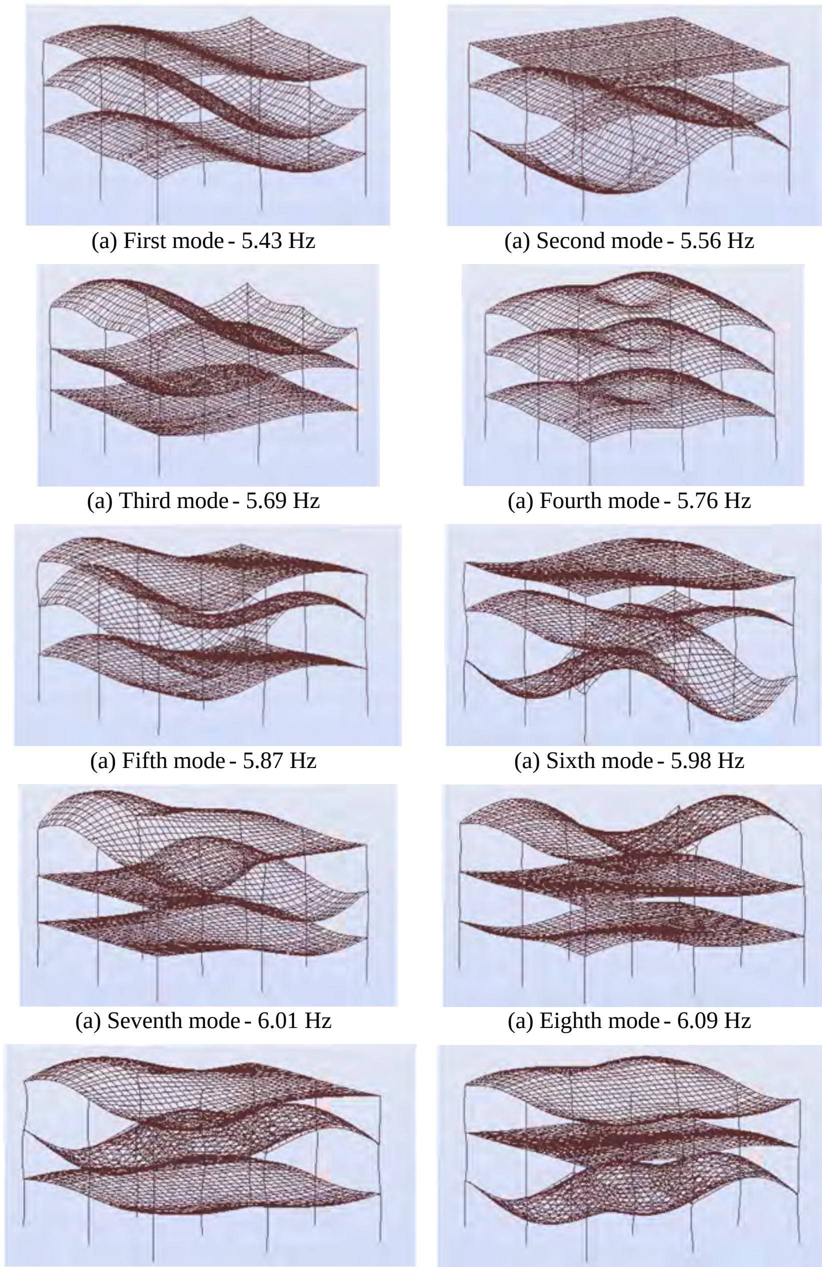
In contrast, the phase coefficient CD is considered in the LM-I analysis, based on a certain number of individuals performing aerobics on the floor and later extrapolated for a larger number of people. This dynamic loading model (LM-I) generated more realistic peak acceleration values and is in agreement with experimental results obtained by Faísca [9]. As mentioned before, relevant variations which lead to the reduction of the dynamic loading on the floor, such as phase lags between the individuals and change of rhythm during the activity are already embedded in this dynamic loading model (LM-I). Therefore, the numerical results will be presented only based on the LM-I formulation, as illustrated in Fig. 10 and Tables 8–13.

Based on the results presented in Fig. 9, it was possible to verify that the dynamic actions coming from aerobics, represented by the dynamic loading models LM-I and LM-II, have generated peak accelerations lower than the acceleration limit of 0.5% g ($a_{\lim} = 490$ mm/s^2) [7,8]: LM-I: $a_p = 218$ mm/s^2 < $a_{lim} = 490$ mm/s^2 and LM-II: $a_p = 390$ mm/s^2 < $a_{lim} = 490$ mm/s^2. This trend was confirmed in several other situations, where the human comfort criterion was not violated in this particular floor. When the non-excited panels were considered in the investigation (see Fig. 6 and Table 5), the peak accelerations presented lower values and the human comfort criterion was satisfied as well when adjacent panels were investigated, as illustrated in Fig. 10.

On the other hand, based on the results presented in Tables 8–13, it is possible to verify that aerobics represented by load model I (LM-I) presented peak accelerations higher than 5% g [7,8] when the composite floors with a main span of 10.0 m ($a_p = 515.9$ mm/s^2 > $a_{lim} = 490$ mm/s^2, see Table 8) and 6.7 m ($a_p = 562.4$ mm/s^2 > $a_{lim} = 490$ mm/s^2, see Table 10) were considered in the analysis, as presented in Tables 8 and 10. This fact can be explained by the proximity between the frequencies of the second harmonic of the dynamic excitation and the fundamental frequency of these investi-

Table 7
Natural frequencies for the different structural models.

Geometries (m × m)	Composite floors finite element natural frequencies (Hz)										AISC [8]
	f_{01}	f_{02}	f_{03}	f_{04}	f_{05}	f_{06}	f_{07}	f_{08}	f_{09}	f_{10}	f_{01}
10 × 10	4.42	4.49	4.62	4.71	4.75	4.85	4.99	5.00	5.08	5.20	4.30
8 × 8	5.43	5.56	5.69	5.76	5.87	5.98	6.01	6.09	6.17	6.30	5.26
6.7 × 6.7	4.77	4.85	4.99	5.30	5.45	5.50	5.63	5.83	5.93	5.98	4.50
5.7 × 5.7	7.21	7.75	8.11	8.24	8.43	8.91	8.96	9.10	9.15	9.24	7.06
5 × 5	7.82	8.21	8.4	8.59	8.74	9.28	9.51	9.75	9.80	9.84	7.67
4 × 4	7.29	8.44	8.6	9.56	9.73	10.05	10.23	10.53	11.11	11.21	7.13



(a) First mode - 5.43 Hz
(a) Second mode - 5.56 Hz
(a) Third mode - 5.69 Hz
(a) Fourth mode - 5.76 Hz
(a) Fifth mode - 5.87 Hz
(a) Sixth mode - 5.98 Hz
(a) Seventh mode - 6.01 Hz
(a) Eighth mode - 6.09 Hz
(a) Ninth mode - 6.17 Hz
(a) Tenth mode - 6.30 Hz

Fig. 8. Vibration modes for the 8 × 8 m composite floor.

gated steel–concrete composite floors characterising the resonance phenomena.

When the peak accelerations of the analysed structural systems with main spans of 8 × 8 m, 5.7 × 5.7 m, 5 × 5 m and 4 × 4 m are considered in the study, the results were quite different, and the human comfort criterion was not violated ($a_p < a_{lim}$). This difference is due to the lack of proximity between the frequencies of dynamic excitation and the natural frequencies of these floors, as presented in Tables 9 and 11–13.

However, based on the results illustrated in Tables 8–10, it can be noticed that, for the human rhythmic activities (aerobics), in the analysed composite floors with main spans of 10 × 10 m, 8 × 8 m and 6.7 × 6.7 m the peak accelerations values are greater than 1.5% g (a_{lim} = 147 mm/s^2) [7,8], and the human comfort criterion was not satisfied in several design situations when adjacent floor panels were investigated (see Tables 8–10). On the other hand, smaller dimensions steel–concrete composite floors (main spans of 5.7 × 5.7 m, 5 × 5 m and 4 × 4 m) did not violate this criterion, as presented in Tables 11–13.

This fact shows that these human rhythmic activities may generate peak accelerations that violated the design criteria when human comfort is considered, even in floor locations without dynamic loadings that are adjacent to loaded floors.

However, it must be emphasised that if the adopted design acceleration limit is considered equal to that in the previous situations (a_{lim} = 490 mm/s^2) [7,8], the situation is clearly different and the human comfort criterion is not violated.

6.3. Dynamic deflections

When performing dynamic analysis of floors subjected to dynamic loadings, a critical parameter may be the dynamic

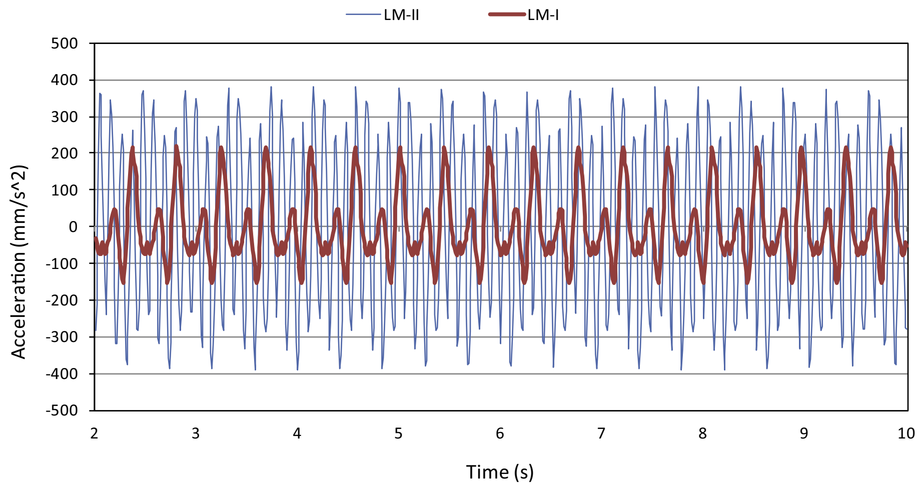
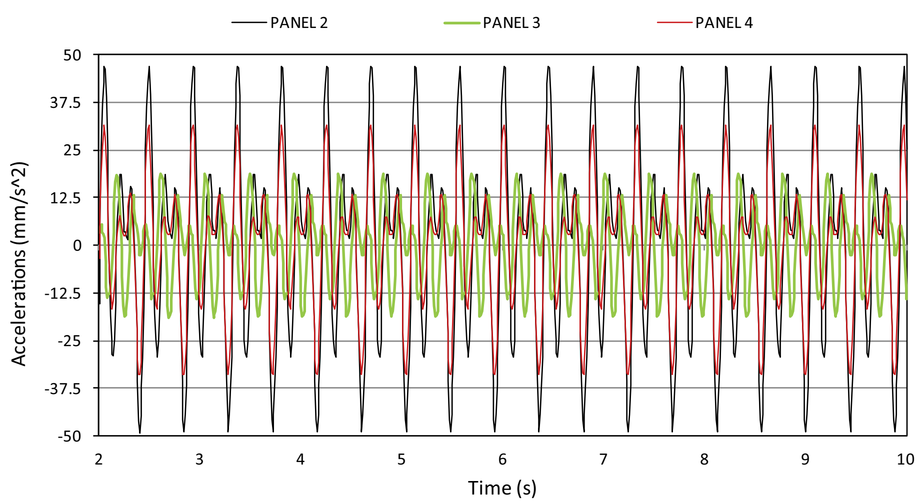
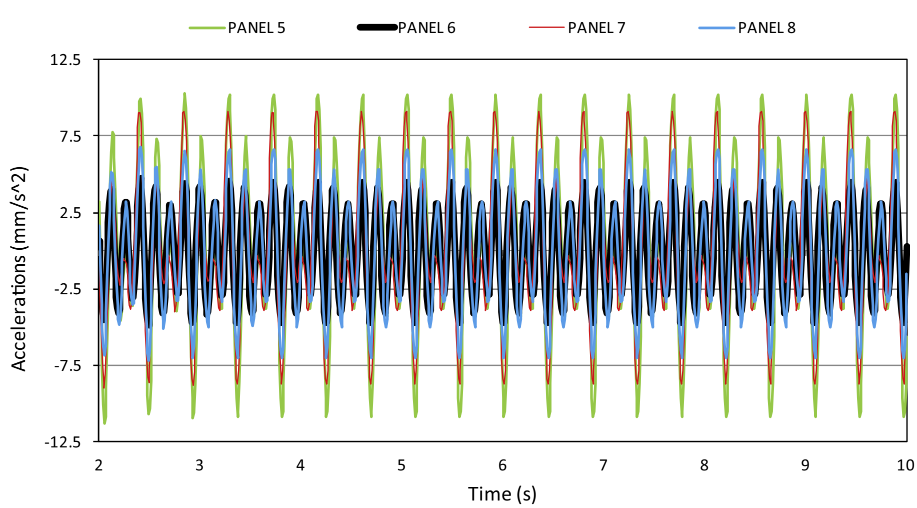


Fig. 9. Composite floor accelerations for the excited panel (8 × 8 m model).



(a) Unloaded panels at the first storey.



(a) Unloaded panels at the second storey.

Fig. 10. Composite floor accelerations at the non-loaded panels (8 × 8 m model).

Table 8
Composite floor peak accelerations: 10 × 10 m structure.

Acceleration in panel a_p (mm/s^2)	Investigated load case (see Tables 5, 6, Figs. 5 and 6)									
	1	2	3	4	5	6	7	8	9	10
1	438.4	331	433.9	465.7	280.8	9.4	15.9	16	29.2	50.4
2	268.7	334.6	384	251.6	282.1	28.6	18	57.8	32.7	50.5
3	179.6	57.6	434.1	245.2	280.8	11.8	35.1	16	25.8	50.5
4	151.6	64.1	384.3	471.3	282.2	29.3	33.7	57.9	18.4	50.6
5	9.5	22.1	15.9	23	50.5	456.6	376.3	426.2	513.2	329
6	23.1	13.1	57.9	27.3	50.8	275.6	378.4	372.7	251	330
7	12.2	30.8	15.9	29.6	50.5	174.2	49.2	426.3	250.3	329.1
8	35.8	39.6	57.9	24.8	50.8	143.3	51.5	372.9	515.9	330.3

a_{lim} = 490 mm/s^2: recommended limit for rhythmic activities [7,8].
a_{lim} = 147 mm/s^2: recommended limit for shopping malls [7,8].

Table 9
Composite floor peak accelerations: 8 × 8 m structure.

Acceleration in panel a_p (mm/s^2)	Investigated load case (see Tables 5, 6, Figs. 5 and 6)									
	1	2	3	4	5	6	7	8	9	10
1	218	205.9	224.2	205	203	11.2	15.8	20.3	18.0	31.7
2	49.1	207.3	83.0	52.4	204.4	6.3	15.8	12.1	16.4	31.6
3	18.8	36.1	224.3	51.6	203.1	9.1	15.8	20.3	13.7	31.7
4	33.8	37.2	83	206	204.5	5.7	15.8	12.1	15.1	31.6
5	11.2	17.3	20.3	16.6	31.7	218.9	205.3	228.5	205.4	201.2
6	4.8	14.3	12	15	31.6	42	208.0	72.4	39.2	202.4
7	9.1	14.5	20.3	15	31.6	19.2	27.1	228.5	38.7	201.3
8	7.2	17.3	12	16.6	31.6	30.8	27.8	72.4	207.8	202.4

a_{lim} = 490 mm/s^2: recommended limit for rhythmic activities [7,8].
a_{lim} = 147 mm/s^2: recommended limit for shopping malls [7,8].

Table 10
Composite floor peak accelerations: 6.7 × 6.7 m structure.

Acceleration in panel a_p (mm/s^2)	Investigated load case (see Tables 5, 6, Figs. 5 and 6)									
	1	2	3	4	5	6	7	8	9	10
1	393.2	252.5	526.6	283.6	265.8	20.9	17	38.1	19	31.4
2	164.2	254.2	287.2	31.5	266.4	14.7	17.3	28.4	16.2	31.8
3	136.5	23.8	526.5	31.2	265.7	17.5	15.6	38.1	13.3	31.5
4	122.9	23.4	286.9	285	266.5	13.7	15.6	28.4	16	31.9
5	20.9	18.7	38.1	17.4	31.4	415.4	265.8	562.4	301.1	297.4
6	14	15.6	28.4	14.7	31.9	169.5	267.5	296.4	18.2	299.2
7	17.5	14.2	38.2	14.7	31.4	153.1	35.8	562.3	18.6	297.3
8	14.4	17.1	28.4	17.7	31.9	127.5	35.5	296	302.7	299.4

a_{lim} = 490 mm/s^2: recommended limit for rhythmic activities [7,8].
a_{lim} = 147 mm/s^2: recommended limit for shopping malls [7,8].

Table 11
Composite floor peak accelerations: 5.7 × 5.7 m structure.

Acceleration in panel a_p (mm/s^2)	Investigated load case (see Tables 5, 6, Figs. 5 and 6)									
	1	2	3	4	5	6	7	8	9	10
1	85.6	77.3	86.5	82	75.2	11.2	17.1	17.4	15.9	28
2	13.8	77.4	18.9	21.4	75.3	10	17.2	14.7	16.5	28.3
3	11.8	12.2	86.5	21.5	75.2	10.3	15	17.4	16.2	28
4	13.8	12.2	18.9	82	75.3	8.8	15.2	14.7	16	28.3
5	8.3	15.2	15.5	13.8	27.9	87.5	84.6	95.8	88.6	94.8
6	6.9	15.2	12.5	14.4	28.4	22.6	84.8	24.9	32.2	95.2
7	7.3	12.8	15.5	14.2	27.9	32.6	33.1	95.8	31.8	94.7
8	5.7	13.1	12.5	14	28.3	24.3	33.3	24.8	88.7	95.1

a_{lim} = 490 mm/s^2: recommended limit for rhythmic activities [7,8].
a_{lim} = 147 mm/s^2: recommended limit for shopping malls [7,8].

Table 12
Composite floor peak accelerations: 5 × 5 m structure.

Acceleration in panel a_p (mm/s^2)	Investigated load case (see Tables 5, 6, Figs. 5 and 6)									
	1	2	3	4	5	6	7	8	9	10
1	78.3	76.4	84.7	75.1	75.4	5.2	8.3	8.8	7.5	14.1
2	9.5	75.8	16.8	11.1	75.4	2.8	8.7	5.3	5.3	13.7
3	17.1	8.2	84.6	11.7	75.5	3.6	5.9	8.8	6.6	14.1
4	11.2	8.6	16.8	75.8	75.1	2.7	5.1	5.3	8.4	13.7
5	4.8	7.5	8.3	7.1	13.3	83.6	83.5	91	81.8	87.3
6	3	8.6	5	5.3	12.9	8.6	82.8	10.8	17.9	86.8
7	3.8	5.8	8.3	6.2	13.3	21	12.7	90.9	19.2	87.4
8	2	5.1	5	7.7	12.9	9.9	12.1	10.8	82	86.5

a_{lim} = 490 mm/s^2: recommended limit for rhythmic activities [7,8].
a_{lim} = 147 mm/s^2: recommended limit for shopping malls [7,8].

Table 13
Composite floor peak accelerations: 4 × 4 m structure.

Acceleration in panel a_p (mm/s^2)	Investigated load case (see Tables 5, 6, Figs. 5 and 6)									
	1	2	3	4	5	6	7	8	9	10
1	40	36	45.4	38.6	41.4	2.8	4	5.3	3.8	7.8
2	4.1	35.6	5.4	11.8	41.4	1.6	4	2.9	4.5	8.1
3	10.1	8.4	45.4	11.7	41.4	2.7	3.8	5.3	4.1	7.8
4	4.1	8.5	5.5	38	41.4	1.3	4	2.8	3.8	8.1
5	2.7	4.1	5.3	3.8	7.8	40.5	38.6	47.7	40.7	45.2
6	1.6	4	2.8	4.3	8.1	3.6	38	4.7	13.7	45
7	2.7	3.7	5.3	4.2	7.8	11.4	10.5	47.7	14	45.2
8	1.5	4.1	2.8	3.8	8.1	4	10.1	4.7	40.6	45

a_{lim} = 490 mm/s^2: recommended limit for rhythmic activities [7,8].
a_{lim} = 147 mm/s^2: recommended limit for shopping malls [7,8].

deflections of the structural system. In the case of floors these dynamic deflections, when downwards, add to the deflections due to the quasi-permanent loading, and the relevant displacements (static, dynamic or a combination of both) should be checked against the relevant code provisions.

As explained in Section 4, the investigated structural models were checked in the design phase in accordance to the ultimate and serviceability limit states codes provisions [4,5,41]. Dynamic deflections were never a concern either, and the highest values observed in the whole range of the studied structural systems are briefly referred in this section. These highest values correspond to the 10 × 10 m span structure (i.e. where the main girders span 10 m and that is the most flexible from all the structures in Table 4), that also exhibits the highest acceleration values as discussed in the previous section (Table 8). For this structural system two load cases are critical:

The first is load case # 8 (see Table 5), where panels 5 and 7 are loaded as shown in Table 5 and in Fig. 6. The time history of the dynamic deflections for this structure is shown in Fig. 11, where the dynamic deflections for the two most representative points in this structural system are plotted against the time. These two points are observation points 7 and 8 in Fig. 6b, respectively on a loaded and on a non-loaded panel. Fig. 11 shows that the maximum dynamic deflections in a loaded panel are 0.5 mm in

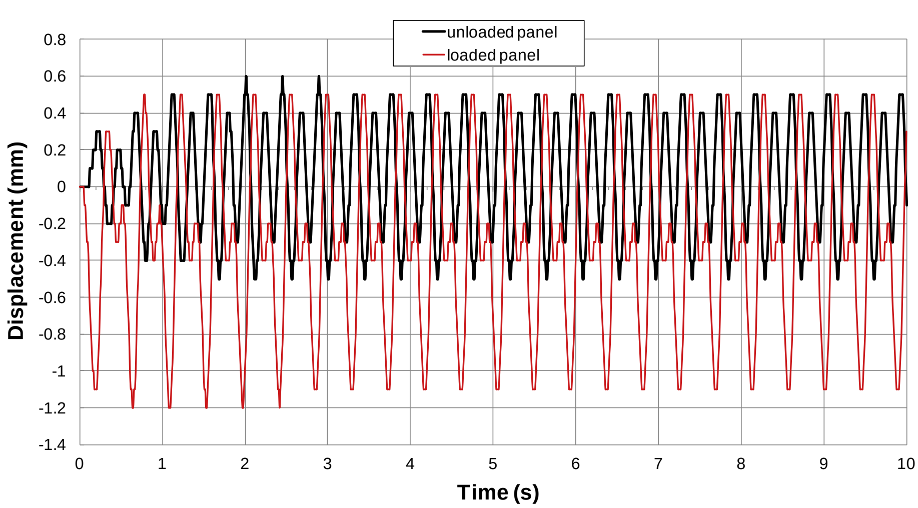


Fig. 11. Composite floor maximum deflections (10 × 10 m model) at the loaded panel 5 and non-loaded panel 6 for load case 8.

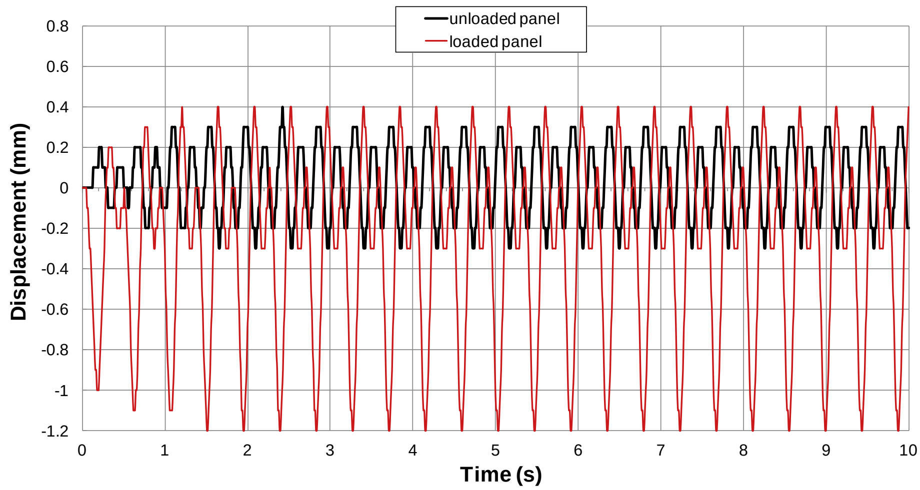


Fig. 12. Composite floor maximum deflections (10 × 10 m model) at the loaded panel 5 and non-loaded panel 6 for load case 9.

the upward direction and 1.2 mm in the downward direction. In an unloaded panel these maximum dynamic deflections are 0.6 mm and 0.5 mm, respectively.

The second load case (# 9 in Table 5) corresponds to dynamic loading in panels 5 and 8 (Table 5 and Fig. 6). Fig. 12 illustrates the time history displacement of the most critical points within a loaded panel (observation point 8 – Fig. 6b) and within a non-loaded panel (observation point 6 – Fig. 6b). The maximum displacements in a dynamic loaded panel are 0.4 mm (upward direction) and 1.2 mm (downward direction). For the non-loaded panels these displacements are 0.4 mm and 0.3 mm, respectively.

It may be concluded that these dynamic displacements lie well below the usual limiting values of span/250, even when the maximum static displacement for the quasi-permanent loading (13.1 mm) is added.

7. Conclusions

This paper contributed to the evaluation of the structural behaviour of composite floors subjected to dynamic excitations induced by human rhythmic activities. The investigation was conducted based on a more realistic load model incorporating the phase coefficient variation CD and considering a certain number of individuals, later extrapolated for a larger number of people.

In this particular load model, the dynamic actions generated by human rhythmic activities, such as jumping, aerobics, and dancing, were investigated based on the results achieved through a long series of experimental tests with individuals performing rhythmic and non-rhythmic activities.

The dynamic behaviour of common composite floors in multi-storey, multi-bay buildings in terms of serviceability limit states was assessed and discussed. The investigated structural models were based on a real steel–concrete composite floor with varying spans for sake of generality of 4 × 4 m, 5 × 5 m, 5.7 × 5.7 m, 6.7 × 6.7 m, 8 × 8 m, and 10 × 10 m, with a total area varying from 16 m^2 to 100 m^2 per panel. The studied structural system consisted of a typical composite floor of a commercial building, where the floors are supported by steel columns and are currently subjected to human rhythmic loads. The models are comprised of composite girders and a 100-mm-thick concrete slab.

The proposed computational model adopted the usual mesh refinement techniques present in finite element method simulations based on the Robot program and enabled a complete dynamic evaluation of the composite floors, especially in terms of human comfort and its associated vibration serviceability limit states.

The composite floor's dynamic response in terms of peak accelerations was obtained and compared to the limits proposed by the ISO [7] and AISC [8]. The results have shown that human rhythmic activities (aerobics) could induce common steel–concrete composite floors designed according to the usual ULS and SLS related to cracking and deformation criteria to reach unacceptable vibration levels and, in these situations, lead to a violation of the current human comfort criteria for these structures.

The obtained results also indicated that in all analysed adjacent floor panels, concerning aerobics, the peak accelerations were higher than the design recommendation acceptable limit; thus, the human comfort criterion was not satisfied in several design situations.

Assessment of the dynamic displacements in the analysed structures showed that these remained within the acceptable limits for all the studied cases, even when added to the static displacements for quasi-permanent loading.

The results obtained in this study have clearly shown the importance of further investigation considering other design parameters, such as structural connections stiffness, floor thickness, structural damping, and the cross-section geometrical properties of beams and columns.

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